

# Geometrical Effects of Baryon Density Inhomogeneities on Primordial Nucleosynthesis

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## ABSTRACT

We discuss effects of fluctuation geometry on primordial nucleosynthesis. For the first time we consider condensed cylinder and cylindrical-shell fluctuation geometries in addition to condensed spheres and spherical shells. We find that a cylindrical shell geometry allows for an appreciably higher baryonic contribution to be the closure density ( $\Omega_b h_{50}^2 \lesssim 0.2$ ) than that allowed in spherical inhomogeneous or standard homogeneous big bang models. This result, which is contrary to some other recent studies, is due to both geometry and recently revised estimates of the uncertainties in the observationally inferred primordial light-element abundances. We also find that inhomogeneous primordial nucleosynthesis in the cylindrical shell geometry can lead to significant Be and B production. In particular, a primordial beryllium abundance as high as  $[\text{Be}] = 12 + \log(\text{Be}/\text{H}) \approx -3$  is possible while still satisfying all of the light-element abundance constraints.

*Subject headings:* cosmology: theory - dark matter - early universe - nuclear reactions, nucleosynthesis, abundances

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## 1. INTRODUCTION

The analysis of primordial nucleosynthesis provides valuable limits on cosmological and particle physics parameters through a comparison between the predicted and inferred primordial abundances of D,  $^3\text{He}$ ,  $^4\text{He}$ , and  $^7\text{Li}$ . For standard homogeneous big bang nucleosynthesis (HBBN) the predicted primordial abundances of these light-elements are in accord with the value inferred from observation provided that baryon-to-photon ratio ( $\equiv \eta$ ) is between about  $2.5 \times 10^{-10}$  and  $6 \times 10^{-10}$ . This corresponds to an allowed range for the baryon fraction of the universal closure density  $\Omega_b^{\text{HBBN}}$  (Walker et al. 1991; Smith, Kawano, & Malaney 1993; Copi, Schramm & Turner 1995; Schramm & Mathews 1995),

$$0.04 \lesssim \Omega_b^{\text{HBBN}} h_{50}^2 \lesssim 0.08, \quad (1)$$

where  $\eta = 6.6 \times 10^{-9} \Omega_b h_{50}^2$ . The lower limit on  $\Omega_b^{\text{HBBN}}$  arises mainly from the upper limit on the deuterium plus  $^3\text{He}$  abundance (Yang et al. 1984; Walker et al. 1991; Smith, Kawano, & Malaney 1993), and the upper limit to  $\Omega_b$  arises from the upper limit on the  $^4\text{He}$  mass fraction  $Y_p$  and/or the deuterium abundance  $D/H \geq 1.2 \times 10^{-5}$  (Linsky et al. 1993, 1995). Here,  $h_{50}$  is the Hubble constant in units of  $50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . The fact that this range for  $\Omega_b h_{50}^2$  is so much greater than the current upper limit to the contribution from luminous matter  $\Omega_b^{\text{Lum}} \lesssim 0.01$  (see however Jedamzik, Mathews & Fuller 1995) is one of the strongest arguments for the existence of baryonic dark matter.

Over the years HBBN has provided strong support for the standard, hot big bang cosmological model as mentioned above. However, as the astronomical data have become more precise in recent years, a possible conflict between the predicted abundances of the light element isotopes from HBBN and the abundances inferred from observations has been suggested (Olive & Steigman 1995; Steigman 1996a; Turner et al. 1996; Hata et al. 1996; see also Hata et al. 1995).

There is now a good collection of abundance information on the  $^4\text{He}$  mass fraction,  $Y_p$ , O/H, and N/H in over 50 extragalactic HII regions (Pagel et al. 1992; Pagel 1993; Izotov, Thuan & Lipovetsky 1994; Skillman & Kennicutt 1995). In an extensive study based upon these observations, the upper limit to  $\eta$  from the observed  $^4\text{He}$  abundance was found to be  $\sim 3.5 \times 10^{-10}$  (Olive & Steigman 1995; Olive & Scully 1996) when a systematic error in  $Y_p$  of  $\Delta Y_{sys} = 0.005$  is adopted. Recently, it has been recognized that the  $\Delta Y_{sys}$  may even be factor of 2 or 3 larger (Thuan, Nature, Izotov, & Lipovetsky 1996; Copi, Schramm & Turner 1995; Schramm & Mathews 1995; Sasselov & Goldwirth 1995), making the upper limit to  $\eta$  as large as  $7 \times 10^{-10}$ .

On the other hand, the lower bound to  $\eta$  has been derived directly from the upper bound to the combined abundances of D and  $^3\text{He}$ . This is because it is believed that

deuterium is largely converted into  $^3\text{He}$  in stars; the lower bound then applies if, as has generally been assumed, a significant fraction of  $^3\text{He}$  survives stellar processing (Walker et al. 1991).

However, there is mounting evidence that low mass stars destroy  $^3\text{He}$  (Wasserburg, Boothroyd & Sackmann al 1995; Charbonnel 1995), although it is possible that massive stars produce  $^3\text{He}$ . Therefore, the uncertainties of chemical evolution models render it difficult to infer the primordial deuterium and  $^3\text{He}$  abundances by using observations of the present interstellar medium (ISM) or from the solar meteoritic abundances. Recent data and analysis lead to a lower bound of  $\eta \gtrsim 3.5 \times 10^{-10}$  on the basis of D and  $^3\text{He}$  (Dearborn, Steigman, & Tosi 1996; Hata et al. 1996; Steigman 1996a; Steigman & Tosi 1995), if the fraction of  $^3\text{He}$  that survives stellar processing in the course of galactic evolution exceeds 1/4. This poses a potential conflict between the observation ( $Y_p$  with low  $\Delta Y_{sys}$ , D) and HBBN.

In this context, possible detections (Songaila et al. 1994; Carswell, Science., et, al. 1994; 1996; Tytler & Fan 1994; Tytler, Fan, & Burlers 1996; Rugers & Hogan 1996a, 1996b; Wampler et al. 1996) of an isotope-shifted Lyman- $\alpha$  absorption line at high redshift ( $z \gtrsim 3$ ) along the line of sight to quasars are of considerable interest. Quasar absorption systems can sample low metallicity gas at early epochs where little destruction of D should have occurred. Thus, they should give definitive measurements of the primordial cosmological D abundance. A very recent high resolution detection by Rugers & Hogan (1996a) suggests a ratio D/H of

$$\text{D/H} = 1.9 \pm 0.4 \times 10^{-4}. \quad (2)$$

This result is consistent with the estimates made by Songaila et al. (1994) and Carswell et al. (1994), using lower resolution. It is also similar to that found recently in another absorption system by Wampler et al. (1996), but it is inconsistent with high resolution studies in other systems at high redshift (Tytler, Fan & Burles 1996; Burles & Tytler 1996) and with the local observations of D and  $^3\text{He}$  in the context of conventional models of stellar and Galactic evolution (Edmunds 1994; Gloeckler & Geiss 1996). If the high value of D/H is taken to be the primordial abundance, then the consistency between the observation and HBBN is recovered and the allowed range of  $\Omega_b$  inferred from HBBN changes to  $\Omega_b^{\text{HBBN}} h_{50}^2 = 0.024 \pm 0.002$  (Jedamzik, Fuller & Mathews 1994; Krauss & Kernan 1994; Vangioni-Flam & Casse 1995). In this case, particularly if  $h_{50}$  is greater than  $\sim 1.5$ , the big bang prediction could be so close to the baryonic density in luminous matter that little or no baryonic dark matter is required (Persic & Salucci 1992; Jedamzik, Mathews & Fuller 1995). This could be in contradiction with observation, particularly if the recently detected microlensing events (Alcock et al. 1993, 1994, 1995abc; Aubourg et al. 1993) are shown to be baryonic. This low baryonic density limit would also be contrary to evidence (White et

al. 1993; White and Fabian 1995) that baryons in the form of hot X-ray gas may contribute a significant fraction of the closure density

The observations by Tytler, et al. (1996) and Burles & Tytler (1996) yield a low value of D/H. Their average abundance is

$$\text{D/H} = 2.4 \pm 0.9 \times 10^{-5}, \quad (3)$$

with  $\pm 2\sigma$  statistical error and  $\pm 1\sigma$  systematic error. This value is consistent with the expectations of local galactic chemical evolution. However, this value would imply an HBBN helium abundance of  $Y_p = 0.249 \pm 0.003$  which is only marginally consistent with the observationally inferred  $Y_p$  even if the high  $\Delta Y_{sys}$  is adopted.

With this in mind, it is worthwhile to consider alternative cosmological models. One of the most widely investigated possibilities is that of an inhomogeneous density distribution at the time of nucleosynthesis. Such studies were initially motivated by speculation (Witten 1984; Applegate & Hogan 1985) that a first order quark-hadron phase transition (at  $T \sim 100$  MeV) could produce baryon inhomogeneities as baryon number was trapped within bubbles of shrinking quark-gluon plasma. In previous calculations using the baryon inhomogeneous big bang nucleosynthesis (IBBN) model, it has been usually assumed that the geometry of baryon density fluctuations is approximated by condensed spheres. Such geometry might be expected to result from a first order QCD phase transition in the limit that the surface tension dominated the evolution of shrinking bubbles of quark-gluon plasma. However, the surface tension may not be large (Kajantie, Kärkkäinen & Rummukainen 1990, 1991, 1992) during the QCD transition, which could lead to a "shell" geometry or the development of dendritic fingers (Freese & Adams 1990). Furthermore, such fluctuations might have been produced by a number of other processes operating in the early universe (cf. Malaney & Mathews 1993), for which other geometries may be appropriate, e.g. strings, sheets, etc. Thus, the shapes of any cosmological baryon inhomogeneities must be regarded as uncertain.

The purpose of this paper is, therefore, to explore the sensitivity of the predicted elemental abundances in IBBN models to the geometry of the fluctuations. We consider here various structures and profiles for the fluctuations in addition to condensed spheres. Mathews et al. (1990, 1994, 1996) found that placing the fluctuations in spherical shells rather than condensed spheres allowed for lower calculated abundances of  $^4\text{He}$  and  $^7\text{Li}$  for the same  $\Omega_b$ , and that a condensed spherical geometry is not necessarily the optimum. Here we show that a cylindrical geometry also allows for an even higher baryonic contribution to the closure density than that allowed by the usually adopted condensed sphere. It appears to be a general result that shell geometries allow for a slightly higher baryon density. This

we attribute to the fact that, for optimum parameters, shell geometries involve a larger surface area to volume ratio and hence more efficient neutron diffusion.

An important possible consequence of baryon inhomogeneities at the time of nucleosynthesis may be the existence of unique nucleosynthetic signatures. Among the possible observable signatures of baryon inhomogeneities already pointed out in previous works are the high abundances of heavier elements such as beryllium and boron (Boyd & Kajino 1989; Kajino & Boyd 1990; Malaney & Fowler 1989; Terasawa & Sato 1990; Kawano et al. 1991), intermediate mass elements (Kajino, Mathews & Fuller 1990), or heavy elements (Malaney & Fowler 1988; Applegate, Hogan & Scherrer 1988; Rauscher et al. 1994). Such possible signatures are also constrained, however, by the light-element abundances. It was found in several previous calculations that the possible abundances of synthesized heavier nuclei was quite small (e.g., Alcock et al. 1990; Terasawa & Sato 1990; Rauscher et al. 1994). We find, however, that substantial production of heavier elements may nevertheless be possible in IBBN models with cylindrical geometry.

## 2. BARYON DENSITY INHOMOGENEITIES

After the initial suggestion (Witten 1985) of QCD motivated baryon inhomogeneities it was quickly realized (Applegate & Hogan 1985; Applegate, Hogan & Scherrer 1987) that the abundances of primordial nucleosynthesis could be affected. A number of papers have addressed this point (Alcock, Fuller, & Mathews 1987; Applegate, Hogan & Scherrer 1987, 1988; Fuller, Mathews, & Alcock 1988; Kurki-Suonio et al. 1988, 1990; Terasawa & Sato 1989a, b, c, 1990; Kurki-Suonio & Matzner 1989, 1990; Mathews et al. 1990, Mathews, Schramm & Meyer 1993; Mathews, Kajino & Orito 1996; Jedamzik, Fuller & Mathews 1994; Jedamzik, Mathews & Fuller 1995; Thomas et al. 1994; Rauscher et al. 1994). Most recent studies in which the coupling between the baryon diffusion and nucleosynthesis has been properly accounted for (e.g., Terasawa & Sato 1989a, b, c, 1990; Kurki-Suonio & Matzner 1989, 1990; Mathews et al. 1990, Mathews, Schramm & Meyer 1993; Jedamzik, Fuller & Mathews 1994; Thomas et al. 1994) have concluded that the upper limit on  $\Omega_b h^2$  is virtually unchanged when compared to the upper limit on  $\Omega_b h^2$  derived from standard HBBN. It is also generally believed (e.g. Vangioni-Flam & Casse 1995) that the same holds true if the new high D/H abundance is adopted.

However, in the previous studies, it was usually assumed that a fluctuation geometry of centrally condensed spheres produces the maximal impact on nucleosynthesis. Here we emphasize that condensed spheres are not necessarily the optimal nor the most physically motivated fluctuation geometry.

Several recent lattice QCD calculations (Kajantie, Kärkkäinen & Rummukainen 1990, 1991, 1992; Brower et al. 1992) indicate that the surface tension of nucleated hadron bubbles is relatively low. In this case, after the hadron bubbles have percolated, the structure of the regions remaining in the quark phase may not form spherical droplets but rather sheets or filaments. We do note that the significant effects on nucleosynthesis may require a relatively strong first order phase transition and sufficient surface tension to generate an optimum separation distance between baryon fluctuations (Fuller, Mathews, & Alcock 1988). However, even if the surface tension is low, the dynamics of the coalescence of hadron droplets may lead to a large separation between regions of shrinking quark-gluon plasma. Furthermore, even though lattice QCD has not provided convincing evidence for a strongly first order QCD phase transition (e.g., Fukugita & Hogan 1991), the order of the transition must still be considered as uncertain (Gottlieb 1991; Petersson 1993). It depends sensitively upon the number of light quark flavors. The transition is first order for three or more light flavors and second order for two. Because the  $s$  quark mass is so close to the transition temperature, it has been difficult to determine the order of transition. At least two recent calculations (Iwasaki et al. 1995; Kanaya 1996) indicate a clear signature of a first order transition when realistic  $u, d, s$  quark masses are included, but others indicate either second order or no phase transition at all.

In addition to the QCD phase transition, there remain a number of alternative mechanisms for generating baryon inhomogeneities prior to the nucleosynthesis epoch (cf. Malaney & Mathews 1993), such as electroweak baryogenesis (Fuller et al. 1994), inflation-generated isocurvature fluctuations (Dolgov & Silk 1993), and kaon condensation (Nelson 1990). Cosmic strings might also induce baryon inhomogeneities through electromagnetic (Malaney & Butler 1989) or gravitational interactions.

Since the structures, shapes, and origin of any baryon inhomogeneities are uncertain, a condensed spherical geometry is not necessarily the most physically motivated choice. Indeed, we will show that a condensed spherical geometry is also not necessarily the optimum to allow for the highest values for  $\Omega_b$  while still satisfying the light-element abundance constraints. Here we consider the previously unexplored cylindrical geometry. String geometries may naturally result from various baryogenesis scenarios such as superconducting axion strings or cosmic strings. Also, the fact that QCD is a string theory may predispose QCD-generated fluctuations to string-like geometry (Kajino & Tassie 1993; Tassie & Kajino 1993). Hence, cylindrical fluctuations may be a natural choice.

### 3. OBSERVATIONAL CONSTRAINTS

We adopt the following constraints on the observed helium mass fraction  $Y_p$  and  ${}^7\text{Li}$  taken from Balbes et al. (1993), Schramm & Mathews (1995), Copi et al. (1995) and Olive (1996):

$$0.226 \leq Y_p \leq 0.247, \quad (4)$$

$$0.7 \times 10^{-10} \leq {}^7\text{Li}/\text{H} \leq 3.5 \times 10^{-10}. \quad (5)$$

This primordial  ${}^4\text{He}$  abundance constraint includes a statistical uncertainty of  $\pm 0.003$  and possible systematic errors as much as  $+0.01/ - 0.005$  with central value of 0.234. A recent reinvestigation (with new data) of the linear regression method for estimating the primordial  ${}^4\text{He}$  abundance has called into question the systematic uncertainties assigned to  $Y_p$  (Izotov, Thuan & Lipovetsky 1996). Our adopted upper limit to  $Y_p$  of Eq. (4) is essentially equal to the limit derived in their study with  $1\sigma$  statistical error.

The upper limit to the lithium abundance adopted here includes the systematic increase from the model atmospheres of Thorburn (1994) and the possibility of as much as a factor of 2 increase due to stellar destruction. This is consistent with the recent observations of  ${}^6\text{Li}$  in halo stars (Smith, Lambert & Nissen 1992; Hobbs & Thorburn 1994). We note that recent discussion of model atmospheres (Kurucz 1995) suggests that as much as an order magnitude upward shift in the primordial lithium abundance could be warranted due to the tendency of one-dimensional models to underestimate the ionization of lithium. Furthermore, a recent determination of the lithium abundance in the globular cluster M92 having the metallicity  $[\text{Fe}/\text{H}] = -2.25$  has indicated that at least one star out of seven shows  $[\text{Li}] = 12 + \log(\text{Li}/\text{H}) \approx 2.5$  (Boesgaard 1996b). Since the abundance measurement of the globular cluster stars is more reliable than that of field stars, this detection along with the possible depletion of lithium in stellar atmospheres suggests that a lower limit to the primordial abundance is  $3.2 \times 10^{-10} \leq {}^7\text{Li}/\text{H}$ .

There also remains the question as to why several stars which are in all respects similar to the other stars in the Population II ‘lithium plateau’, are so lithium rich or lithium deficient (Deliyannis et al. 1995; Boesgaard 1996a, 1996b). Until this is clarified, it may be premature to assert that the Population II abundance of lithium reflects the primordial value. The primordial abundance may instead correspond to the much higher value observed in Population I stars which has been depleted down to the Population II lithium plateau. The observational evidence (Deliyannis, Pinsonneault & Duncan 1993) for a  $\pm 25\%$  dispersion in the Population II lithium plateau is consistent with this hypothesis (Deliyannis et al. 1993; Charbonnel 1995; Steigman 1996b). Rotational depletion was studied in detail by Pinsonneault et al. (1992) who note that the depletion factor could

have been as large as 10. Chaboyer and Demarque (1994) also demonstrated that models incorporating both rotation and diffusion provide a good match to the observed  ${}^7\text{Li}$  depletion with decreasing temperature in Population II stars and their model indicated that the initial lithium abundance could have been as high as  ${}^7\text{Li}/\text{H} = 1.23 \pm 0.28 \times 10^{-9}$ .

A recent study (Ryan et al. 1996), which includes new data on 7 halo dwarfs, fails to find evidence of significant depletion through diffusion, although other mechanisms are not excluded. For example, stellar wind-driven mass loss could deplete a high primordial lithium abundance of down to the Population II value [Eq. (5)] in a manner consistent with  ${}^6\text{Li}$  observations (Vauclair & Charbonnel 1995). Furthermore, it could be possible (Yoshii, Mathews, & Kajino 1995) that some of the  ${}^6\text{Li}$  is the result of more recent accretion of interstellar material that could occur as halo stars episodically plunge through the disk. Such a process could mask the earlier destruction of lithium. For comparison, therefore, we adopt a conservative upper limit on the primordial lithium abundance of

$${}^7\text{Li}/\text{H} < 1.5 \times 10^{-9}. \quad (6)$$

Finally, the primordial abundance of deuterium is even harder to clarify since it is easily destroyed in stars (at temperatures exceeding about  $6 \times 10^5\text{K}$ ). Previously, limits on the deuterium (and also the  ${}^3\text{He}$ ) abundances have been inferred from their presence in presolar material (e.g., Walker et al. 1991). It is also inferred from the detection in the local interstellar medium (ISM) through its ultraviolet absorption lines in stellar spectra (McCullough 1992; Linsky et al. 1993, 1995). The limit from ISM data is consistent with that from abundances in presolar material. It has been argued that there are no important astrophysical sources of deuterium (Epstein et al. 1976) and ongoing observational attempts to detect signs of deuterium synthesis in the Galaxy are so far consistent with this hypothesis (see Pasachoff & Vidal-Madjar 1989). If this is indeed so, then the lowest D abundance observed today should provide a lower bound to the primordial abundance. Recent precise measurements by Linsky et al. (1995, 1993) using the *Hubble Space Telescope* implies

$$\text{D}/\text{H} > 1.2 \times 10^{-5}. \quad (7)$$

We adopt this as a lower limit to the primordial deuterium abundance for the purposes of exploring the maximal cosmological impact from IBBN. In addition, we consider the two possible detections of the deuterium abundance along the line of sight to high red shifted quasars, Eqs. (2) and (3) as possible limits.

In order to derive a lower limit to  $\Omega_b h_{50}^2$ , it is useful to consider the sum of deuterium plus  ${}^3\text{He}$ . In the context of a closed-box instantaneous recycling approximation, it is straightforward (Olive et al. 1990) to show that the sum of primordial deuterium and  ${}^3\text{He}$



can be written

$$y_{23p} \leq A_{\odot}^{(g_3-1)} y_{23\odot} \left( \frac{X_{\odot}}{X_p} \right) \quad (8)$$

where  $A_{\odot}$  is the fraction of the initial primordial deuterium still present when the solar system formed,  $g_3$  is the fraction of  $^3\text{He}$  that survives incorporation into a single generation of stars,  $y_{23\odot}$  is the presolar value of  $[\text{D}+^3\text{He}]/\text{H}$  inferred from the gas rich meteorites, and  $X_{\odot}/X_p$  is the ratio of the presolar hydrogen mass fraction to the primordial value. These factors together imply an upper limit (Walker et al. 1991; Copi, Schramm & Turner 1995) of

$$y_{23p} \leq 1.1 \times 10^{-4}. \quad (9)$$

#### 4. CALCULATIONS

The calculations described here are based upon the coupled diffusion and nucleosynthesis code of Mathews et al. (1990), but with a number of nuclear reaction rates updated and the numerical diffusion scheme modified to accommodate cylindrical geometry. We also have implemented an improved numerical scheme which gives a more accurate description of the effects of proton and ion diffusion, and Compton drag at late times. Although our approach is not as sophisticated as that of Jedamzik et al. (1994a), it produces essentially the same results for the parameters employed here. We have also included all of the new nuclear reaction rates summarized in Smith et al. (1993) as well as those given in Thomas et al. (1993). We obtain the same result as Smith et al. (1993) using these rates and homogeneous conditions in our IBBN model

Calculations were performed in a cylindrical geometry both with the high density regions in the center (condensed cylinders), and with the high density regions in the outer zone of computation (cylindrical shells). Similarly, calculations were made in a spherical geometry with the high density regions in the center (condensed spheres) and with the high density region in the outer zones of computation (spherical shells).

In the calculations, the fluctuations are resolved into 16 zones of variable width as described by Mathews et al. (1990). We assumed three neutrino flavors and an initially homogeneous density within the fluctuations. Such fluctuation shapes are the most likely to emerge, for example, after neutrino-induced expansion (Jedamzik & Fuller 1994). We use a neutron mean life-time of  $\tau_n = 887.0$  (Particle Data Group 1994). In addition to the cosmological parameter,  $\Omega_b$  and fluctuation geometry, there remain three parameters to specify the baryon inhomogeneity. They are:  $R$ , the density contrast between the high and low-density regions;  $f_v$ , the volume fraction of the high-density region; and  $r$ , the average

separation distance between fluctuations.

## 5. RESULTS

The parameters  $R$  and  $f_v$  were optimized to allow for the highest values for  $\Omega_b h_{50}^2$  while still satisfying the light-element abundance constraints. For fluctuations represented by condensed spheres, optimum parameters are  $R \sim 10^6$  and  $f_v^{1/3} \sim 0.5$  (Mathews, Kajino & Orito 1996). For other fluctuation geometries, we have found that optimum parameters are:

$$R \sim 10^6; \quad \text{for all fluctuation geometries}$$

$$f_v^{1/3} \sim 0.19; \quad \text{for spherical shells}$$

$$f_v^{1/2} \sim \begin{cases} 0.5; & \text{for condensed cylinders} \\ 0.15; & \text{for cylindrical shells,} \end{cases}$$

although there is not much sensitivity to  $R$  once  $R \gtrsim 10^3$ . Regarding  $f_v$ , we have written the appropriate length scale of high density regions, i.e.  $f_v^{1/3}$  and  $f_v^{1/2}$  for the spherical and cylindrical fluctuation geometries, respectively. The variable parameters in the calculation are then the fluctuation cell radius  $r$ , and the total baryon-to-photon ratio  $\eta$  (or  $\Omega_b h_{50}^2$ ).

### 5.1. Constraints on $\Omega_b h_{50}^2$

Figures 1, 2, 3(a), and 4(a) show contours of allowed parameters in the  $r$  versus  $\eta$  and  $r$  versus  $\Omega_b h_{50}^2$  plane for the adopted light-element abundance constraints of Eqs. (4) - (6) and for a possible Lyman- $\alpha$  D/H of Eqs. (2) and (3), for the condensed sphere, spherical shell, condensed cylinder, and cylindrical shell fluctuation geometries, respectively. The fluctuation cell radius  $r$  is given in units of meters for a comoving length scale fixed at a temperature of  $kT = 1$  MeV. Both of the possible  ${}^7\text{Li}$  limits, Eqs. (5) and (6) which we have discussed above, are also drawn as indicated. In order to clearly distinguish the two abundance constraints, we use the single and double-cross hatches for the regions allowed by the adopted lower (Eq. (5)) and higher (Eq. (6)) limits to the  ${}^7\text{Li}$  primordial abundance.

Even in the IBBN scenario, if the low D/H of Eq. (3) (Burles & Tytler 1996) is adopted as primordial, this range for D/H appears to be compatible with the  ${}^7\text{Li}$  abundance only when a higher (Population I) primordial  ${}^7\text{Li}$  abundance limit is adopted, except for a very

narrow region of  $\eta \sim 6 \times 10^{-10}$  and  $r \leq 10^2 m$ . This conclusion remains unchanged for any other fluctuation geometries. Therefore, the acceptance of the low (Burles & Tytler 1996) value of D/H would strongly suggest that significant depletion of  ${}^7\text{Li}$  has occurred.

In contrast, adoption of the high D/H of Eq. (2) (Rugers & Hogan 1996a) as primordial allows the concordance of all light-elements. The upper limits to  $\eta$  and  $\Omega_b h_{50}^2$  are largely determined by D and  ${}^7\text{Li}$ . The concordance range for the baryon density is comparable to that for HBBN for small separation distance  $r$ . However, there exist other regions of the parameter space with optimum separation distance, which roughly corresponds to the neutron diffusion length during nucleosynthesis (Mathews et al. 1990), with an increased maximum allowable value of the baryonic contribution to the closure density to  $\Omega_b h_{50}^2 \leq 0.05$  for the cylindrical geometry, as displayed in Fig. 4(a). This is similar to the value for spherical shells as shown in Mathews et al. (1996) and also in Fig. 2 in the present work. The condensed sphere limits, however, are essentially unchanged from those of the HBBN model. If the primordial  ${}^7\text{Li}$  abundance could be as high as the upper limit of  $\text{Li}/\text{H} \leq 1.5 \times 10^{-9}$ , the maximum allowable value of the baryonic content for the condensed sphere would increase to  $\Omega_b h_{50}^2 \leq 0.08$ , with similar values for the spherical shell (Mathews, Kajino & Orito 1996). For both the condensed cylinders and cylindrical shells, the upper limits could be as high as  $\Omega_b h_{50}^2 \leq 0.1$  as shown in Figs. 3(a) and 4(a). These higher upper limits relative to those of the HBBN are of interest since they are consistent with the inferred baryonic mass in the form of hot X-ray gas (White et al. 1993; White and Fabian 1995) in dense galactic clusters. The acceptance of this consistence, as noted above, requires the significant stellar depletion of  ${}^7\text{Li}$ .

In Figures 3(b) and 4(b), we also show contours for the condensed cylinder and cylindrical shell geometries, respectively, but this time with the conventional light-element constraints of Eqs. (4), (5), (7), and (9) as indicated. Since the results for the condensed sphere and spherical shell geometries with this set of the conventional abundance constraints have already been discussed by Mathews et al. (1996), we do not show those contours here. The cylindrical shell geometry of the present work gives the highest allowed value of  $\Omega_b h_{50}^2$ . Figure 4(b) shows that the upper limits to  $\eta$  and  $\Omega_b h_{50}^2$  are largely determined by  $Y_p$  and  ${}^7\text{Li}$ . The upper limits for a cylindrical shell geometry could be as high as  $\Omega_b h_{50}^2 \leq 0.13$  with similar results for the spherical shell geometry (Mathews, Kajino & Orito 1996). A high primordial lithium abundance would increase the allowable baryonic content to as high as  $\Omega_b h_{50}^2 \leq 0.2$ . The reason that shell geometries allow for higher baryon densities we attribute to more efficient neutron diffusion which occurs when the surface area to volume area is increased. This allows for more initial diffusion to produce deuterium, and more efficient back diffusion to avoid over producing  ${}^7\text{Li}$ .

## 5.2. Observational Signature

The production of beryllium and boron as well as lithium in IBBN models can be sensitive to neutron diffusion. Therefore, their predicted abundances are sensitive to not only the fluctuation parameter  $r$ ,  $R$ , and  $f_v$  but also the fluctuation geometry (Boyd & Kajino 1989; Malaney & Fowler 1989; Kajino & Boyd 1990; Terasawa & Sato 1990). Figures 5 - 7 show the contours of the calculated abundances for lithium, beryllium and boron, respectively in the  $r$  versus  $\eta$  (and  $r$  versus  $\Omega_b h_{50}^2$ ) plane. the shaded region depicts allowed values of  $r$  and  $\eta$  from the light element abundance constraints [cf. Fig. 4(b)] for a cylindrical shell fluctuation geometry. The contour patterns of lithium (Fig. 5) and boron (Fig. 7) abundances are very similar, whereas there is no similarity found between lithium (Fig. 5) and beryllium (Fig. 6) abundances.

In order to understand the similarities and differences among these three elemental abundances, we show in Figs. 8 and 9 the decompositions of the  $A = 7$  abundance into  ${}^7\text{Li}$  and  ${}^7\text{Be}$  and the boron abundance into  ${}^{10}\text{B}$  and  ${}^{11}\text{B}$ . These Figures show also the dependence of the predicted LiBeB abundances in IBBN on the scale of fluctuations for a cylindrical shell geometry with fixed  $\Omega_b h_{50}^2 = 0.1$ . This value of  $\Omega_b h_{50}^2$  corresponds to a typical value in the allowable range of  $\eta$  in Fig. 4(b), which optimizes the light element abundance constraints, even satisfying the lower  ${}^7\text{Li}$  abundance limit of Eq. (5). The fluctuation parameters  $f_v$  and  $R$  are the same as in Fig. 4(b). Once the baryonic content  $\Omega_b$  is fixed, the only variable parameter is the separation distance,  $r$ .

As can be seen in Fig. 8, as the separation  $r$  increases, neutron diffusion plays an increasingly important role in the production of  $t$  and, by the  ${}^4\text{He}(t, \gamma){}^7\text{Li}$  reaction. It works maximally around  $r \sim 10^4$  m, which is the typical length scale of neutron diffusion at  $kT = 1$  MeV. A similar behavior is observed in the  ${}^7\text{Li}(t, n){}^9\text{Be}$  reaction. This reaction produces most of the  ${}^9\text{Be}$  in neutron rich environments where  $t$  and  ${}^7\text{Li}$  are abundant, as was first pointed out by Boyd and Kajino (1989). At other separation distances  $r$  in a  $\Omega_b h_{50}^2 = 0.1$  model, most of the  $A = 7$  nuclides are created as  ${}^7\text{Be}$  by the  ${}^4\text{He}({}^3\text{He}, \gamma){}^7\text{Be}$  reaction. In the limit of  $r =$  horizon scale, the nucleosynthesis products are approximately equal to the sum of those produced in the proton-rich and neutron-rich zones separately (Jedamzik, Fuller, Mathews & Kajino 1994). The predominant contribution from the proton-rich zones makes the  ${}^7\text{Be}$  abundance almost constant at larger  $r$ , while both  ${}^7\text{Li}$  and  ${}^9\text{Be}$  decrease as  $r$  increases toward the horizon at any separation distance.

Figure 9 shows that  ${}^{11}\text{B}$  is a predominant component of the total boron abundance at any separation distance. This is true for almost all values  $\Omega_b h_{50}^2$ . It has been pointed out (Malaney & Fowler 1988; Applegate, Hogan & Scherrer 1988; Kajino & Boyd 1990) that most  ${}^{11}\text{B}$  is produced by the  ${}^7\text{Li}(n, \gamma){}^8\text{Li}(\alpha, n){}^{11}\text{B}$  reaction sequence in neutron-rich

environments at relatively early times when most of the other heavier nuclides are made. Recent measurements of the previously unmeasured  ${}^7\text{Li}(\alpha, n){}^{11}\text{B}$  reaction cross section (Boyd et al. 1992; Gu et al. 1995; Boyd, Paradellis & Rolfs 1996) at the energies of cosmological interest have removed the significant ambiguity in the calculated  ${}^{11}\text{B}$  abundance due to this reaction. The factor of two discrepancy among several different measurements of the reaction cross section for  ${}^7\text{Li}(n, \gamma){}^8\text{Li}$  was also resolved by the new measurement (Nagai et al. 1991). The  ${}^7\text{Li}(\alpha, \gamma){}^{11}\text{B}$  reaction also makes an appreciable but weaker contribution to the production of  ${}^{11}\text{B}$  in the neutron-rich environment. In the proton-rich environment, on the other hand, the  ${}^7\text{Be}(\alpha, \gamma){}^{11}\text{C}$  reaction contributes largely to the production of  ${}^{11}\text{C}$  which beta decays to  ${}^{11}\text{B}$  in 20.39 min. These facts explain why the contour patterns of the lithium and boron abundances in Figs. 5 and 7 look very similar.

It is conventional in the literature to quote the beryllium and boron abundance relative to  $\text{H} = 10^2$ . Hence, one defines the quantity  $[\text{X}] = 12 + \log(\text{X}/\text{H})$ . In cylindrical shell fluctuation geometry the beryllium abundance can take the value of  $[\text{Be}] \sim -3$  while still satisfying all of the light-element abundance constraints and the Population II lithium abundance constraint (Figs. 5 and 6). This abundance is higher by three orders magnitude than that produced in the HBBN model with conventional light-element abundance constraints. This result is contrary to a recent result with the condensed sphere geometry and for a more restricted parameter space (Thomas et al. 1994). Recent beryllium observation of Population II stars (Rebolo et al. 1988; Ryan et al. 1990, 1992; Ryan 1996; Gilmore et al. 1992a, 1992b; Boesgaard & King 1993; Boesgaard 1994, 1996a,b) have placed the upper limit on the primordial  ${}^9\text{Be}$  abundance to  $[\text{Be}] \sim -2$ , one order magnitude greater than the beryllium abundance in the IBBN cylindrical model.

The calculated boron abundance at the optimum separation distance is essentially equal to the value of the HBBN model. However, a high primordial lithium abundance would increase the upper limit to  $\Omega_b h_{50}^2$ . In this case, the boron abundance could be one or two orders magnitude larger than that of the HBBN model (Fig. 7).

## 6. CONCLUSIONS

We have reinvestigated the upper limit to  $\eta$  and  $\Omega_b h_{50}^2$  in inhomogeneous primordial nucleosynthesis models. We have considered effects of various geometries. In particular, for the first time we consider cylindrical geometry. We have also incorporated recently revised light-element abundance constraints including implications of the possible detection (Songaila et al. 1994; Carswell, Science., et, al. 1994; 1996; Tytler & Fan 1994; Tytler, Fan, & Burlers 1996; Rugers & Hogan 1996a, 1996b; Wampler et al. 1996) of a high deuterium

abundance in Lyman- $\alpha$  absorption systems. We have shown that with low primordial deuterium (Tytler & Fan 1994; Tytler, Fan, & Burlers 1996), significant depletion of  $^7\text{Li}$  is required to obtain concordance between predicted light-element abundance of any model of BBN and the observationally inferred primordial abundance. If high primordial deuterium (Rugers & Hogan 1996a) is adopted (Eq. (2)), there is a concordance range which is largely determined by D/H, and the upper limit to  $\Omega_b h_{50}^2$  is 0.05. However, with the presently adopted (Eqs. (4), (6), (7), (9)) light-element abundance constraints (Schramm & Mathews 1995; Copi, Schramm & Turner 1995; Olive & Scully 1996), values of  $\Omega_b h_{50}^2$  as large as 0.2 are possible in IBBN models with cylindrical-shell fluctuation geometry.

We have also found that significant beryllium and boron production is possible in IBBN models without violating the light element abundance constraints. The search for the primordial abundance of these elements in low metallicity stars could, therefore, be a definitive indicator of the presence or absence of cylindrical baryon inhomogeneities in the early universe.

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Fig. 1.— Contours of allowed values for baryon-to-photon ratio  $\eta$  (or  $\Omega_b h_{50}^2$ ) and fluctuation separation radius  $r$  based upon the various light-element abundance constraints as indicated. The separation  $r$  is given in units of meters comoving at  $kT = 1$  MeV. This calculation is based upon baryon density fluctuations represented by condensed spheres. The cross hatched region is allowed by the adopted primordial abundance limits with high (Eq. (2)) and low (Eq. (3)) deuterium abundance in Lyman limit systems and also a higher extreme  ${}^7\text{Li}$  upper limit (Eq. (6)). The single hatched region depicts the allowed parameters for lower  ${}^7\text{Li}$  (Eq. (5)) constraint. Note that the  ${}^7\text{Li}$  abundance is the sum of  ${}^7\text{Li}$  and  ${}^7\text{Be}$ .

Fig. 2.— Same as Fig. 1, but for fluctuations represented by spherical shells.

Fig. 3(a).— Same as Fig. 1, but for fluctuations represented by condensed cylinders. Adopted primordial deuterium abundance constraints are inferred from observations of Lyman limit systems (Eqs. (2) and (3)).

Fig. 3(b).— Same as Fig. 1, but for fluctuations represented by condensed cylinders. Adopted primordial deuterium and  ${}^3\text{He}$  abundance constraints are inferred from observations of ISM (Eqs. (7) and (9)).

Fig. 4(a).— Same as Fig. 3(a), but for fluctuations represented by cylindrical shells.

Fig. 4(b).— Same as Fig. 3(b), but for fluctuations represented by cylindrical shells.

Fig. 5.— Contours of the predicted abundance of lithium (the sum of  ${}^7\text{Li}$  and  ${}^7\text{Be}$ ), for baryon-to-photon ratio  $\eta$  (or  $\Omega_b h_{50}^2$ ) and fluctuation separation radius  $r$ , in the cylindrical shell fluctuation geometry. The shaded region displays the allowed  $\eta$ – $r$  region from Fig. 4(b).

Fig. 6.— Same as Fig. 5, but for beryllium,  ${}^9\text{Be}$ .

Fig. 7.— Same as Fig. 5, but for boron,  ${}^{10}\text{B} + {}^{11}\text{B}$ .

Fig. 8.— Lithium and beryllium abundances as function of proper separation distance in units of meters comoving at  $kT = 1$  MeV for fixed  $\Omega_b h_{50}^2 = 0.1$ . Refer to the abundance scales in *l.h.s.* for lithium and *r.h.s.* for beryllium.

Fig. 9.— Same as Fig. 8, but for boron.